A new picture for the chiral symmetry properties within a particle-core framework

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Abstract

The Generalized Coherent State Model, proposed previously for a unified description of magnetic and electric collective properties of nuclear systems, is extended to account for the chiral like properties of nuclear systems. To a phenomenological core described by the GCSM a set of interacting particles are coupled. Among the particle-core states one identifies a finite set which have the property that the angular momenta carried by the proton and neutron quadrupole bosons and the particles respectively, are mutually orthogonal. All terms of the model Hamiltonian satisfy the chiral symmetry except for the spin-spin interaction. The magnetic properties of the particle-core states, where the three mentioned angular momenta are orthogonal, are studied. A quantitative comparison of these features with the similar properties of states where the three angular momenta belong to the same plane is performed.

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I. INTRODUCTION

The rotational spectra appear to be a reflection of a spontaneous rotational symmetry breaking when the nuclear system acquires a static nuclear deformation. The fundamental nuclear properties like nuclear shape, the nucleon mass and charge distributions inside the nucleus, electric and magnetic moments, collective spectra may be evidenced through the system interaction with an electromagnetic field. The two components of the field, electric and magnetic, are used to explore the properties of electric and magnetic nature, respectively. At the end of last century the scissors like states [3, 4] as well as the spin-flip excitations [6] have been widely treated by various groups. Some of them were based on phenomenological assumptions while the other ones on microscopic considerations. The scissors like excitations are excited in (e,e') experiments at backward angles and expected at an energy of about 2-3 MeV, while the spin-flip excitations are seen in (p,p') experiments at forward angles and are located at about 5-10 MeV. The scissors mode describes the angular oscillation of proton against neutron system and the total strength is proportional to the nuclear deformation squared which reflects the collective character of the excitation. Many papers have been written on this subject and therefore it is difficult to quote all of them. We mention however two reviews given in Refs. [5, 6].

Since the total M1 strength of the $M1$ mode is proportional to the nuclear deformation squared, it was believed that the magnetic collective properties are in general associated with deformed systems. This is not true due to the magnetic dipole bands, where the ratio between the moment of inertia and the B(E2) value for exciting the first 2^+ from the ground state 0^+, $\mathcal{I}^{(2)}/B(E2)$, takes large values, of the order of $100(\mathrm{eb})^{-2}MeV^{-1}$. These large values can be justified by a large transverse magnetic dipole moment (perpendicular to the total angular momentum) which induces dipole magnetic transitions, but almost no charge quadrupole moment [1]. Indeed, there are several experimental data showing that the dipole bands have large values for $B(M1) \sim 3-6\mu_N^2$ and very small values of $B(E2) \sim 0.1(eb)^2$ (see for example Ref.[2]). The states are different from the scissors mode, they being rather of a shears character. A system with a large transverse magnetic dipole moment (the component of the magnetic moment perpendicular to the total angular momentum) which was studied in many publications, may consist of a triaxial core to which a proton prolate and a neutron oblate hole orbital are coupled. The interaction of particle and hole like orbitals is repulsive, which keeps the two orbits apart from each other. In this way the orthogonal angular momenta carried by the proton particles and neutron holes are favored. The maximal transverse dipole momentum is achieved, for example, when j_p is oriented along the small axis of the core, j_n along the long axis and the core rotates around the intermediate axis. Suppose the three orthogonal angular momenta form a right trihedral frame. If the Hamiltonian describing the interacting system of protons, neutrons and the triaxial core is invariant to the transformation which changes the orientation of one of the three angular momenta, i.e. the right trihedral frame is transformed to a left type, one says that the system exhibits a chiral symmetry. As always happens, such a symmetry is identified when that is broken and consequently to the two trihedral-s correspond distinct energies, otherwise close to each other. Thus, a signature for a chiral symmetry characterizing a triaxial system is the existence of two $\Delta I = 1$ bands which are close in energies. Increasing the total angular momentum the gradual alignment of $\vec{j}_p$ and $\vec{j}_n$ to the total $\vec{J}$ takes place and a magnetic band is developed.

The question addressed in this paper is whether the picture of the three angular momenta system, carried by a phenomenological core, a prolate and an oblate single particle orbitals, with respect to which the chiral symmetry is defined is unique for determining states connected with large M1 transitions. Note that the nuclear system which accommodate the chiral frame is odd-odd.

In the past, the magnetic states of orbital or of spin-flip nature were considered by our group in several publications [7–16]. We studied also the dipole bands with $K^\pi = 1^\pm$ using a quadrupole and octupole boson Hamiltonian and a set of model states obtained by parity and angular momentum projections from a quadrupole deformed ground state without space reflection symmetry [17]. We pointed out that the band 1^+ has a magnetic character while the dipole band 1^- is of an electric type. In another publication [18] we pointed out that the parity partner bands have the property that starting from a critical angular momentum, the states have the property that the angular momenta carried by the quadrupole and octupole bosons respectively, are mutually orthogonal. Therefore one may expect that adding to the phenomenological Hamiltonian a set of interacting particles one could achieve a configuration where the angular momentum carried by nucleons is perpendicular on the quadrupole and octupole angular momenta which are already orthogonal. The first attempt was already made in Ref.[19].

Here we attempt another chiral system consisting of one phenomenological core with two components, one for protons and one for neutrons, and two quasiparticles whose total angular momentum is oriented along the symmetry axis of the core due to the particle-core interaction. We investigate whether states of total angular momentum $\vec{I}$, where the three components mentioned above carry angular momenta, $\vec{J}_p, \vec{J}_n, \vec{J}$, which are mutually orthogonal, may exist. We believe that if such configuration exists it is optimal for defining large transverse magnetic moment inducing large M1 transitions.

II. THE GENERALIZED COHERENT STATE MODEL

The description of magnetic properties in nuclei has always been a central issue. The reason is that the two systems of protons and neutrons respond differently when they interact with an external electromagnetic field. Differences are due to the fact that by contrast to neutrons, protons are charged particles, the proton and neutron magnetic moments are different from each other and, finally, the proton and neutron numbers in a given nucleus are, in general, different.

Many papers have been devoted to explaining various features of the collective dipole mode called, conventionally, scissors mode. The name of the mode was suggested by Lo Iudice and Palumbo who interpreted the dipole mode, within the Two Rotor Model [3], as a scissors like oscillation of proton and neutron systems described by two axially symmetric ellipsoids, respectively.

The Coherent State Model (CSM), proposed by Raduta *et al.* to describe the lowest three collective interacting bands [20], was extended by including the isospin degrees of freedom in order to account for the collective properties of the scissors mode [21]. This extension is conventionally called "The Generalized Coherent State Model"(GCSM).

CSM starts with the construction of a restricted collective space, by projecting out the components of good angular momentum from three orthogonal quadrupole boson states. These states are chosen such that they are orthogonal before and after projection. One of the three deformed states, the intrinsic ground state, is a coherent state of Glauber type with respect to the zero component of the quadrupole boson, $b^\dagger_{20}$, while the other two are obtained by acting with elementary boson polynomials on the ground state. In choosing the intrinsic excited states we take care that the projected states considered in the vibrational limit have to provide the multi-phonon vibrational spectrum, while for the large deformation regime their behavior coincides with that predicted by the liquid drop model.

In contrast to the CSM, which uses only one boson for the composite system of protons and neutrons, within the GCSM the protons are described by quadrupole proton-like bosons, $b^\dagger_{p\mu}$ while the neutrons by quadrupole neutron-like bosons, $b^\dagger_{n\mu}$. Since one deals with two quadrupole bosons instead of one, one expects to have a more flexible model and to find a simpler solution satisfying the restrictions required by CSM. The restricted collective space is defined by the states describing the three major bands, ground, beta and gamma, as well as the band based on the isovector state 1^+. Orthogonality conditions, required for both intrinsic and projected states, are satisfied by the following 6 functions which generate by angular momentum projection, 6 rotational bands:

$$\begin{aligned}
\phi^{(g)}_{JM} &= N^{(g)}_J P^J_{M0}\psi_g, \quad \psi_g = exp[(d_p b^\dagger_{p0} + d_n b^\dagger_{n0}) - (d_p b_{p0} + d_n b_{n0})]|0\rangle, \\
\phi^{(\beta)}_{JM} &= N^{(\beta)}_J P^J_{M0}\Omega_\beta \psi_g, \\
\phi^{(\gamma)}_{JM} &= N^{(\gamma)}_J P^J_{M2}(b^\dagger_{n2} - b^\dagger_{p2})\psi_g, \\
\tilde{\phi}^{(\gamma)}_{JM} &= \tilde{N}^{(\gamma)}_J P^J_{M2}(\Omega^\dagger_{\gamma,p,2} + \Omega^\dagger_{\gamma,n,2})\psi_g, \\
\phi^{(1)}_{JM} &= N^{(1)}_J P^J_{M1}(b^\dagger_n b^\dagger_p)_{11}\psi_g, \\
\tilde{\phi}^{(1)}_{JM} &= \tilde{N}^{(1)}_J P^J_{M1}(b^\dagger_{n1} - b^\dagger_{p1})\Omega^\dagger_\beta \psi_g.
\end{aligned} \tag{2.1}$$

Here, the following notations have been used:

$$\begin{aligned}
\Omega^\dagger_{\gamma,k,2} &= (b^\dagger_k b^\dagger_k)_{22} + d_k\sqrt{\frac{2}{7}}b^\dagger_{k2}, \quad k = p, n, \\
\Omega^\dagger_\beta &= \Omega^\dagger_p + \Omega^\dagger_n - 2\Omega^\dagger_{pn}, \\
\Omega^\dagger_k &= (b^\dagger_k b^\dagger_k)_0 - \sqrt{\frac{1}{5}}d^2_k, \quad k = p, n, \\
\Omega^\dagger_{pn} &= (b^\dagger_p b^\dagger_n)_0 - \sqrt{\frac{1}{5}}d^2_p. \\
\hat{N}_{pn} &= \sum_m b^\dagger_{pm} b_{nm}, \ \hat{N}_{np} = (\hat{N}_{pn})^\dagger, \quad \hat{N}_k = \sum_m b^\dagger_{km} b_{km}, \ k = p, n.
\end{aligned} \tag{2.2}$$

Note that a priory we cannot select one of the two sets of states $\phi^{(\gamma)}_{JM}$ and $\tilde{\phi}^{(\gamma)}_{JM}$ for gamma band, although one is symmetric and the other asymmetric against proton-neutron permutation. The same is true for the two isovector candidates for the dipole states. In Ref.[22], results obtained by using alternatively a symmetric and an asymmetric structure for the gamma band states were presented. Therein it was shown that the asymmetric structure for the gamma band does not conflict any of the available data. By contrary, considering for the gamma states an asymmetric structure and fitting the model Hamiltonian coefficients in the manner described in Ref.[22], a better description for the beta band energies is obtained. Moreover, in that situation the description of the E2 transition becomes technically very simple. For these reasons, here we make the option for a proton neutron asymmetric gamma band.

All calculations performed so far considered equal deformations for protons and neutrons. The deformation parameter for the composite system is:

$$d = \sqrt{2}d_p = \sqrt{2}d_n. \tag{2.3}$$

The factors N involved in the wave functions are normalization constants calculated in terms of some overlap integrals.

We seek now an effective Hamiltonian for which the projected states (2.1) are, at least in a good approximation, eigenstates in the restricted collective space. The simplest Hamiltonian fulfilling this condition is:

$$\begin{aligned} H &= A_1(\hat{N}_p + \hat{N}_n) + A_2(\hat{N}_{pn} + \hat{N}_{np}) + \frac{\sqrt{5}}{2}(A_1 + A_2)(\Omega^\dagger_{pn} + \Omega_{np}) \\ &\quad + A_3(\Omega^\dagger_p \Omega_n + \Omega^\dagger_n \Omega_p - 2\Omega^\dagger_{pn}\Omega_{np}) + A_4 \hat{J}^2. \end{aligned} \tag{2.4}$$

The Hamiltonian given by Eq.(2.4) has only one off-diagonal matrix element in the basis (2.1). That is $\langle \phi^{\beta}_{JM}|H|\tilde{\phi}^{(\gamma)}_{JM}\rangle$. However, our calculations show that this affects the energies of β and $\tilde{\gamma}$ bands by an amount of a few keV. Therefore, the excitation energies of the six bands are in a very good approximation, given by the diagonal element:

$$E^{(k)}_J = \langle \phi^{(k)}_{JM}|H|\phi^{(k)}_{JM}\rangle - \langle \phi^{(g)}_{00}|H|\phi^{(g)}_{00}\rangle, \quad k = g, \beta, \gamma, 1, \tilde{\gamma}, \tilde{1}. \tag{2.5}$$

It can be easily checked that the model Hamiltonian does not commute with the components of the $\hat{F}$ spin operator:

$$\hat{F}_0 = \frac{1}{2}(\hat{N}_p - \hat{N}_n), \ \hat{F}_+ = \hat{N}_{pn}, \ \hat{F}_- = \hat{N}_{np}. \tag{2.6}$$

Hence, the eigenstates of H are F_0 mixed states. However, the expectation values of the F_0 operator on the projected model states are equal to zero. This is caused by the fact that the proton and neutron deformations are considered to be equal. In this case the states are of definite parity, with respect to the proton-neutron permutation, which is consistent with the structure of the model Hamiltonian which is invariant with respect to such a symmetry transformation. To conclude, by contrast to the IBA2 Hamiltonian, the GCSM Hamiltonian is not $\hat{F}$ spin invariant. Another difference to the IBA2, the most essential one, is that the GCSM Hamiltonian does not commute with the boson number operators. Due to this feature the coherent state approach proves to be the most adequate one to treat the Hamiltonian in Eq.(2.4). The asymptotic behavior of the magnetic state 1^+, derived in Ref.[21], shows clearly that the phenomenological description of two liquid drops and two rigid rotors are just particular cases of the GCSM, defined by specific restrictions.

The GCSM seems to be the only phenomenological model which treats simultaneously the M1 and E2 properties. Indeed, in Refs.[22, 23] the ground, beta and gamma bands are considered together with a $K^\pi = 1^+$ band built on the top of the scissor mode 1^+. By contrast to the other phenomenological and microscopic models, which treat the scissors mode in the intrinsic reference frame, here one deals with states of good angular momentum and therefore there is no need to restore the rotational symmetry. As shown in Ref.[24] the GCSM provides for the total M1 strength an expression which is proportional to the nuclear deformation squared. Consequently, the M1 strength of 1^+ and the B(E2) value for 2^+ are proportional to each other, although the first quantity is determined by the convection current while the second one by the static charge distribution.

One weak point of most phenomenological models is that they use expressions for transition operators not consistent with the structure of the model Hamiltonian. Thus, the transition probabilities are influenced by the chosen Hamiltonian only through the wave functions. By contradistinction in Refs. [22, 23] the E2 transition operator, as well as the M1 form-factor are derived analytically, by using the equation of motion of the collective coordinates determined by the model Hamiltonian. In this way a consistent description of electric and magnetic properties of many nuclei was attained.

III. PROTON AND NEUTRON ANGULAR MOMENTA COMPOSITION OF THE GROUND AND DIPOLE MAGNETIC BANDS

We start by mentioning few properties for the intrinsic ground state wave function.

$$\begin{aligned}\Psi_g &\equiv \Psi_p\Psi_n = \sum_{J_p,J_n=even} C_{J_p}|J_p0\rangle C_{J_n}|J_n0\rangle \\ &= \sum_{J_p,J_n,J} C_{J_p}C_{J_n}C^{J_p\ J_n\ J}_{0\ \ 0\ \ 0}|J,0\rangle. \end{aligned} \tag{3.1}$$

The angular momentum projected state is defined by:

$$\begin{aligned}\varphi^{(g)}_{JM} &= N^{(g)}_J P^J_{M0}\Psi_g = N^{(g)}_J \sum_{J_pJ_n} C_{J_p}C_{J_n}C^{J_p\ J_n\ J}_{0\ \ 0\ \ 0}|J,M\rangle \\ &= N^{(g)}_J \sum_{J_pJ_n}\left(N^{(g)}_{J_p}\right)^{-1}\left(N^{(g)}_{J_n}\right)^{-1} C^{J_p\ J_n\ J}_{0\ \ 0\ \ 0}\left[\varphi^{(g)}_{J_p}\varphi^{(g)}_{J_n}\right]_{JM} \end{aligned} \tag{3.2}$$

In the above equations the standard notation for the Clebsch-Gordan coefficients has been used.

The average value of the angular momentum carried by the proton bosons is given by:

$$\langle\varphi^{(g)}_{JM}|\hat{J}^2_p|\varphi^{(g)}_{JM} = \left(N^{(g)}_J\right)^2\sum_{J_p,J_n}\left(N^{(g)}_{J_p}\right)^{-2}\left(N^{(g)}_{J_n}\right)^{-2}J_p(J_p+1)\left(C^{J_p\ J_n\ J}_{0\ \ 0\ \ 0}\right)^2 \equiv \widetilde{J}^{(g)}_{pJ}(\widetilde{J}^{(g)}_{pJ}+1). \tag{3.3}$$

Similarly one calculates the average angular momentum carried by the neutron bosons, $\widetilde{J}^{(g)}_{nJ}$. The two angular momenta, $\widetilde{J}^{(g)}_{pJ}$, $\widetilde{J}^{(g)}_{nJ}$, define the relative angle which obey the equation:

$$\cos(\vec{J}_p,\vec{J}_n)^{(g)}_J = \frac{J(J+1)-\widetilde{J}^{(g)}_{pJ}(\widetilde{J}^{(g)}_{pJ}+1)-\widetilde{J}^{(g)}_{nJ}(\widetilde{J}^{(g)}_{nJ}+1)}{2\sqrt{\widetilde{J}^{(g)}_{pJ}(\widetilde{J}^{(g)}_{pJ}+1)\widetilde{J}^{(g)}_{nJ}(\widetilde{J}^{(g)}_{nJ}+1)}}. \tag{3.4}$$

Let us consider now the angular momentum projection of following dipole excitation of the intrinsic ground state

$$\begin{aligned}\phi^{(1)}_{JM} &= N^{(1)}_J P^J_{M1}(b^\dagger_n b^\dagger_p)_{11}\psi_g \\ &= N^{(1)}_J \sum_{J'=even}\left(N^{(g)}_{J'}\right)^{-1} C^{J'\ 1\ J}_{0\ \ 1\ 1}\left[\left(b^\dagger_n b^\dagger_p\right)_1\varphi^{(g)}_{J'}\right]_{JM}. \end{aligned} \tag{3.5}$$

with the norm having the expression:

$$\left(N^{(1)}_J\right)^{-2} = \sum_{J'=even}\left(N^{(g)}_{J'}\right)^{-2}\left(C^{J'\ 1\ J}_{0\ \ 1\ 1}\right)^2 \tag{3.6}$$

It is worth calculating the separate contributions of proton and neutron bosons to building up the total angular momentum of a given magnetic dipole state. The effective angular momentum $\tilde{J}$ is defined as:

$$
\begin{aligned}
&\widetilde{J}^{(1)}_{p;J}(\widetilde{J}^{(1)}_{p;J}+1) = \langle\varphi^{(1)}_{JM}|\hat{J}^2_p|\varphi^{(1)}_{JM}\rangle \\
&= 6 + \left(N^{(1)}_J\right)^2 \sum_{J_p,J_n,J'} \left(N^{(g)}_{J_p}\right)^{-2} \left(N^{(g)}_{J_n}\right)^{-2} J_p(J_p+1) \left(C^{J_p\ J_n\ J'}_{0\ 0\ 0}\right)^2 \left(C^{J'\ 1\ J}_{0\ 1\ 1}\right)^2 \qquad (3.7)
\end{aligned}
$$

Since the ground state is symmetric with respect to the $p-n$ permutation, one expects that the effective neutron angular momentum defined by averaging the operator $\hat{J}^2_{n;J}$ with the ground state projected function is equal to the effective proton angular momentum, i.e.

$$
\widetilde{J}^{(1)}_{n;J} = \widetilde{J}^{(1)}_{p;J} \qquad (3.8)
$$

Denoting the ground state angular momentum by

$$
\vec{J}_{pn} = \vec{J}_p + \vec{J}_n, \qquad (3.9)
$$

then for the average value one obtains:

$$
\widetilde{J}^{(1)}_{pn;J}(\widetilde{J}^{(1)}_{pn;J}+1) \equiv \langle\varphi^{(1)}_{JM}|\hat{J}^2_{pn}|\varphi^{(1)}_{JM}\rangle = \left(N^{(1)}_J\right)^2 \sum_{J''} \left(N^{(g)}_{J''}\right)^{-2} \left(C^{J''\ 1\ J}_{0\ 1\ 1}\right)^2 (J''(J''+1)+12)\,. \qquad (3.10)
$$

Squaring Eq.(3.9) and averaging the result with the dipole projected state J one can calculate the angle between the angular momenta J_p and J_n:

$$
\cos(\vec{J}_p,\vec{J}_n)^{(1)}_J = \frac{\widetilde{J}^{(1)}_{pn;J}(\widetilde{J}^{(1)}_{pn;J}+1) - \widetilde{J}^{(1)}_{p;J}(\widetilde{J}^{(1)}_{p;J}+1) - \widetilde{J}^{(1)}_{n;J}(\widetilde{J}^{(1)}_{n;J}+1)}{2\sqrt{\widetilde{J}^{(1)}_{p;J}(\widetilde{J}^{(1)}_{p;J}+1)\widetilde{J}^{(1)}_{n;J}(\widetilde{J}^{(1)}_{n;J}+1)}}. \qquad (3.11)
$$

IV. A POSSIBLE EXTENSION OF THE GCSM

Here we shall consider a particle-core interacting system described by the following Hamiltonian:

$$
\begin{aligned}
H &= H_{GCSM} + \sum_{\alpha} \epsilon_a c^\dagger_\alpha c_\alpha - \frac{G}{4} P^\dagger P \\
&- \sum_{\tau=p,n} X^{(\tau)}_{pc} \sum_m q_{2m}\left(b^\dagger_{\tau,-m} + (-)^m b_{\tau m}\right)(-)^m - X_{sS}\vec{J}_F\cdot\vec{J}_c \qquad (4.1)
\end{aligned}
$$

with the notation for the particle quadrupole operator:

$$
\begin{aligned}
q_{2m} &= \sum_{a,b} Q_{a,b}\left(c^{\dagger}_{j_a}c_{j_b}\right)_{2m},\\
Q_{a,b} &= \frac{\hat{j}_a}{\hat{2}}\langle j_a||r^2Y_2||j_b\rangle
\end{aligned}
\tag{4.2}
$$

Here H_{GCSM} denotes the phenomenological Hamiltonian described in previous section, associated to a proton and neutron bosonic core. The next two terms stand for a set of particles moving in a spherical shell model mean-field and interacting among themselves through pairing interaction. The low indices α denote the set of quantum numbers labeling the spherical single particle shell model states, i.e. $|\alpha\rangle = |nljm\rangle = |a,m\rangle$ The last two terms denoted hereafter as H_{pc} expresses the interaction between the satellite particles and the core through a quadrupole-quadrupole and a spin-spin force, respectively. The angular momenta carried by the core and particles are denoted by $\vec{J}_c(= \vec{J}_{pn}$ and $\vec{J}_F$, respectively. The mean field plus the pairing term is quasi-diagonalized by means of the Bogoliubov-Valatin transformation:

$$
\begin{aligned}
a^{\dagger}_{\alpha} &= U_a c^{\dagger}_{\alpha} - V_a s_{\alpha} c_{-\alpha},\quad s_{\alpha} = (-)^{j_{\alpha}-m_{\alpha}}\\
a_{\alpha} &= U_a c_{\alpha} - V_a s_{\alpha} c^{\dagger}_{-\alpha},\quad (-\alpha) = (a, -m_{\alpha}).
\end{aligned}
\tag{4.3}
$$

The free quasiparticle term is $\sum_{\alpha} E_a a^{\dagger}_{\alpha} a_{\alpha}$ while the qQ interaction preserves the above mentioned form, with the factor q_{2m} changed to:

$$
\begin{aligned}
q_{2m} &= \eta^{(-)}_{ab}\left(a^{\dagger}_{j_a}a_{j_b}\right)_{2m} + \xi^{(+)}_{ab}\left((a^{\dagger}_{j_a}a^{\dagger}_{j_b})_{2m} - (a_{j_a}a_{j_b})_{2m}\right),\quad \text{where}\\
\eta^{(-)}_{ab} &= \frac{1}{2}Q_{ab}\left(U_aU_b - V_aV_b\right),\quad \xi^{(+)}_{ab} = \frac{1}{2}Q_{ab}\left(U_aV_b + V_aU_b\right).
\end{aligned}
\tag{4.4}
$$

We restrict the single particle space to a single-j state where two particles are placed. In the space of the particle-core states we, therefore, consider the basis defined by:

$$
\begin{aligned}
& |BCS\rangle \otimes \varphi^{(1)}_{JM},\\
\Psi^{(2qp;J1)}_{JI;M} &= N^{(2qp;J1)}_{JI}\sum_{J'} C^{J\ J'\ I}_{J\ 1\ J+1}\left(N^{(1)}_{J'}\right)^{-1}\left[(a^{\dagger}_j a^{\dagger}_j)_J|BCS\rangle \otimes \varphi^{(1)}_{J'}\right]_{IM}.
\end{aligned}
\tag{4.5}
$$

where $|BCS\rangle$ denotes the quasiparticle vacuum while N_{JI} is the norm given by

$$
\left(N^{(2qp;J1)}_{JI}\right)^{-2} = \sum_{J'} 2\left(N^{(1)}_{J'}\right)^{-2}\left(C^{J\ J'\ I}_{J\ 1\ J+1}\right)^2.
\tag{4.6}
$$

The matrix elements of the model Hamiltonian H are given analytically in Appendix A.

Now let us analyze the proton and neutron angular momentum composition for the two quasiparticle components of the particle-core basis. The effective angular momenta can be easily calculated:

$$
\begin{aligned}
&\widetilde{J}^{(1)}_{\tau;JI}(\widetilde{J}^{(1)}_{\tau;JI}+1) = \langle \Psi^{(2qp;J1)}_{JI}|\hat{J}^2_{\tau}|\Psi^{(2qp;J1)}_{JI}\rangle \\
= &\left(N^{(2qp;J1)}_{JI}\right)^2 \sum_{J'} 2\left(C^{J\ J'\ I}_{J\ 1\ J+1}\right)^2 \left(N^{(1)}_{J'}\right)^{-2} \widetilde{J}_{\tau;J'}(\widetilde{J}_{\tau;J'}+1),\ \tau=p,n, \\
&\widetilde{J}^{(1)}_{pn;JI}(\widetilde{J}^{(1)}_{(pn;JI}+1) = \langle \Psi^{(2qp;J1)}_{JI}|(\hat{J}_p+\hat{J}_n)^2|\Psi^{(2qp;J1)}_{JI}\rangle \\
= &\left(N^{(2qp;J1)}_{JI}\right)^2 \sum_{J'} 2\left(C^{J\ J'\ I}_{J\ 1\ J+1}\right)^2 \left(N^{(1)}_{J'}\right)^{-2} \widetilde{J}^{(1)}_{pn;J'}(\widetilde{J}^{(1)}_{pn;J'}+1).
\end{aligned}
\tag{4.7}
$$

The angle between proton and neutron angular momenta can be obtained from the equation:

$$
\cos(\vec{J}_p,\vec{J}_n)^{(1)}_{JI} = \frac{\widetilde{J}^{(1)}_{pn;JI}(\widetilde{J}^{(1)}_{pn;JI}+1)-\widetilde{J}^{(1)}_{p;JI}(\widetilde{J}^{(1)}_{p;JI}+1)-\widetilde{J}^{(1)}_{n;JI}(\widetilde{J}^{(1)}_{n;JI}+1)}{2\sqrt{\widetilde{J}^{(1)}_{p;JI}(\widetilde{J}^{(1)}_{p;JI}+1)\widetilde{J}^{(1)}_{n;JI}(\widetilde{J}^{(1)}_{n;JI}+1)}}. \tag{4.8}
$$

V. ABOUT THE CHIRAL SYMMETRY

It is worth studying the separate contribution of protons and neutrons to the total angular momentum of a state belonging to the ground band, to the pure phenomenological dipole band and to two quasiparticle-dipole band, respectively. For the three bands this was analytically given by Eqs. (3.5), (3.6) and (4.7), and plotted in upper, middle and bottom panels of Fig.1 respectively. Therein, the notations $< J^{(g)}_{\tau} >$ stay for $\widetilde{J}^{(1)}_{\tau}$, $\widetilde{J}^{(1)}_{\tau;J}$ and $\widetilde{J}^{(1)}_{\tau;JI}$, respectively. Note that for ground band states, when the the proton and the neutron deformations are equal and large, the two angular momenta are aligned to each other. If the two deformations are very different then by far the largest contribution is brought by the most deformed system. As for the pure phenomenological dipole band we note an even-odd staggering for small and moderate deformation. Such a structure is washed out for large deformation. These features are met also for the case of two quasiparticle-dipole states when the two quasiparticle angular momentum is equal to zero. Due to the large K quantum number of the two quasiparticle components, when the angular momentum carried by two quasiparticles is equal to 12, the dipole band starts with the angular momentum 13.

The two quasiparticle-dipole state components of the particle-core basis involve three angular momenta, $\vec{J}_p, \vec{J}_n, \vec{J}_{pn}=\vec{J}_p+\vec{J}_n$ which could be, in certain states, mutually orthogonal.

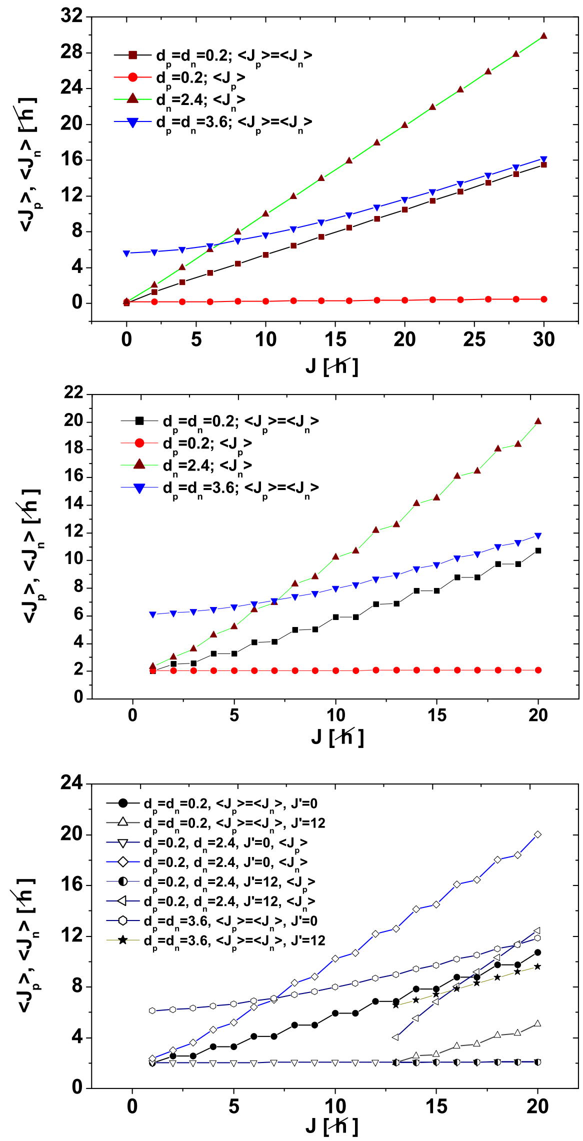

FIG. 1: Proton and neutron angular momentum composition of the states from the ground band (upper panel),the pure phenomenological dipole band (middle panel) and the two quasiparticle-dipole band (bottom panel)

The relative angle of the proton and neutron angular momenta in the pure boson dipole state $\varphi^{(1)}_{JM}$ is presented in Fig.2. One notices that the angle is 90^0 in the first three dipole states of angular momenta 1,2 and 3. Increasing the total spin, the corresponding angles

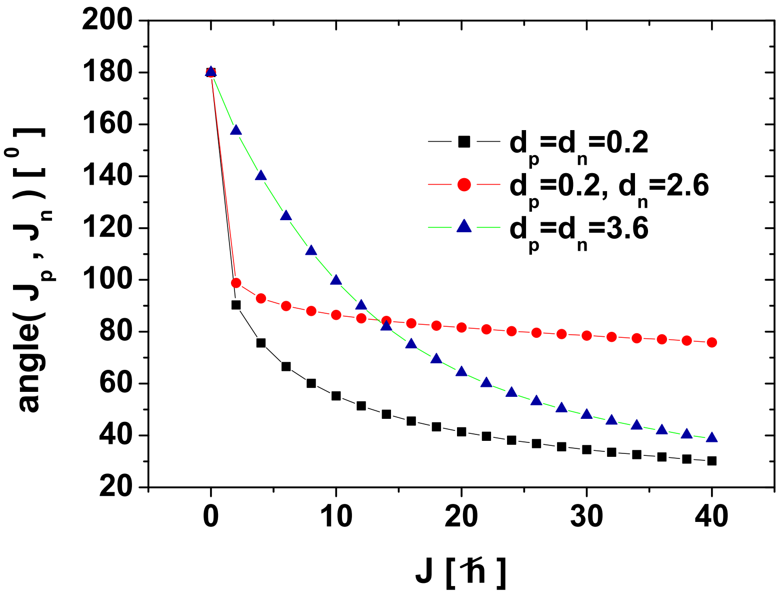

FIG. 2: The angle between $\vec{J}_p$ and $\vec{J}_n$ within the ground-band states $\Phi^{(g)}_{JM}$ for three sets of deformations (d_p, d_n).

decrease monotonically. A step structure for the states J and $J+1$ with J-even shows up.

Note that the unprojected state ψ_g is defined for equal deformation parameters for the proton and neutron systems. However since the number of protons and neutrons are different and moreover the two kinds of nucleons occupy different shells it is reasonable to suppose different deformation parameters for protons and neutrons respectively. The corresponding projected states are denoted by $\Phi^{(1)}_{JM}(d_p, d_n)$. The dependence of the $(\vec{J}_p, \vec{J}_n)$ angle on the total angular momentum is presented in Fig. 3.

When the deformation for protons is different from that of neutrons the step structure is diminished and the total angular momenta where the relative angle is about 90^0 are shifted to 5,6 and 7. The angle decreases with angular momentum but with a much lower slope. Indeed, in the considered angular momentum interval the angle varies between 91.5^0 and 87^0.

Let us see, now, how this picture modifies when we add to the boson dipole states the two quasiparticle state factor. As shown in Fig. 4, the case of common small deformation for protons and neutrons is similar to that from Fig. 2 where the two quasiparticle factor

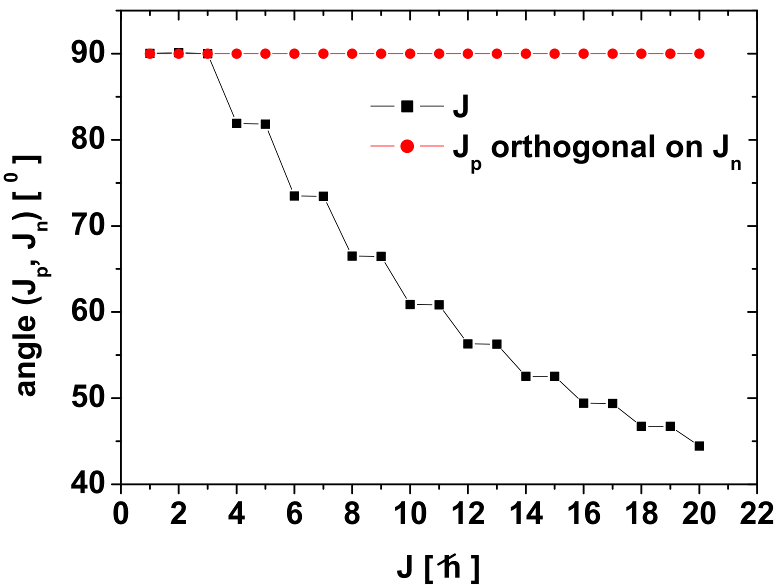

FIG. 3: The angle between $\vec{J}_p$ and $\vec{J}_n$ within the boson dipole state $\varphi^{(1)}_{JM}$. The deformation parameter ρ is equal to 0.2

is missing. By contrast, here we have seven sets of states distinguished by the angular momentum J carried by the quasiparticle component. Otherwise the step function structure as well as the decreasing behavior as function of the total angular momentum are preserved by any of the seven sets. The same remark holds also for Fig. 5 when compared with the situation from Fig.3. Indeed, it seems that the larger the difference between proton and neutron deformations, the smaller the departure of the $(\vec{J}_p, \vec{J}_n)$ angle from 90^0 and the less pronounced the step structure of the angle J-dependence. From Fig. 4 it is clear that for each value of the two quasiparticle angular momentum there are three states, the lowest angular momentum states being characterized by an orthogonal configuration $(\vec{J}_p, \vec{J}_n)$. Since the K quantum numbers for proton and neutron systems included in the core are small, and moreover the total K being equal to unity, it is reasonable to suppose that $\vec{J}_p$ and $\vec{J}_n$ are both perpendicular to the intrinsic symmetry axis, that is OZ. The symmetry axis of the particle motion is determined the mean field caused by the particle core interaction of the qQ type. On the other hand, the quasiparticle angular momentum projection on the symmetry axis is, by construction, maximal. Therefore $\vec{J}_F$ is oriented along the axis

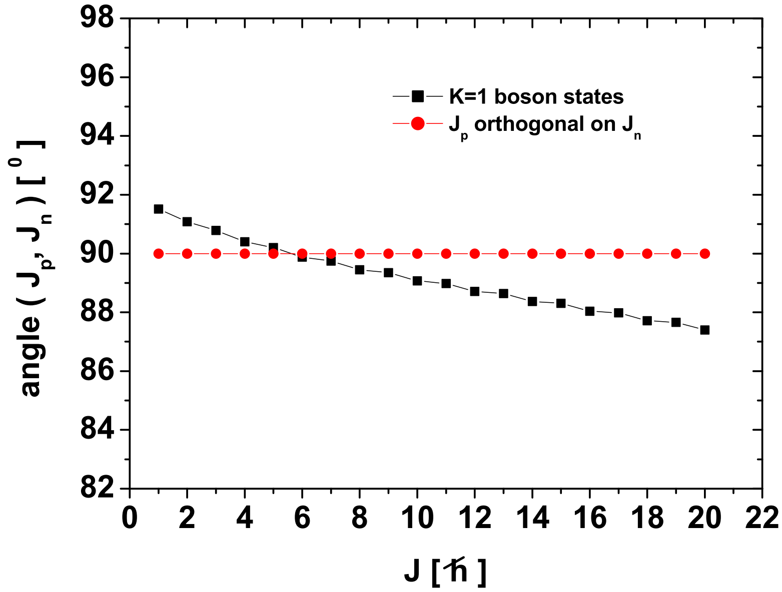

FIG. 4: The angle between $\vec{J}_p$ and $\vec{J}_n$ within the boson dipole state $\Phi^{(1)}_{JM}(d_p, d_n)$. The deformation parameters are $d_p = 0.2$ and $d_n = 2.4$.

OZ, which results in having an orthogonal triedrum $(\vec{J}_p, \vec{J}_n, \vec{J}_F)$. Invoking the arguments of Ref.[1], for such states a large transverse dipole moment is expected, which may induce a large M1 transition rate. *If one ignores the spin-spin interaction term, the resulting Hamiltonian is invariant to changing the orientation of one of the triedrum component, which means that this Hamiltonian exhibits a chiral symmetry. The spin-spin interaction breaks the chiral symmetry and therefore lifts the associated degeneracy. By succesively changing the orientation of one triedhrum componet one obtains three distinct Hamiltonians and therefore one expects four bands. Each of these bands may be related to the remaining three bands by specific chiral transformation, respectively.* These features are in detail studied in what follows.

However, before doing that let us consider the states with the quasiparticle factor state with angular momentum and projection $(J, 0)$:

$$\Psi^{(2qp;01)}_{JI;M} = \mathcal{N}^{(2qp;01)}_{JI} \sum_{J'} C^{J\ J'\ I}_{0\ 1\ 1} \left[(a^\dagger_j a^\dagger_j)_J \varphi^{(1)}_{J'}\right]_{IM} \left(N^{(1)}_{J'}\right)^{-1}. \quad (5.1)$$

In such a state, the three angular momenta, $\vec{J}_p, \vec{J}_n, \vec{J}_F$ are in the same plane. Hence, one

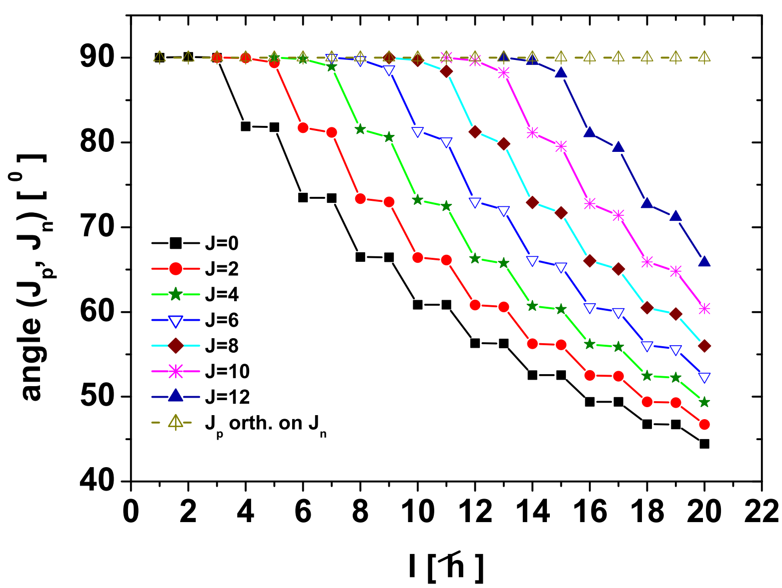

FIG. 5: The angle between $\vec{J}_p$ and $\vec{J}_n$ within the boson dipole state $\varphi^{(2qp;J1)}_{JI;M}(d)$. The deformation parameter ρ is equal to 0.2.

expects the magnetic properties are different from those characterizing the state where the mentioned vectors are mutually orthogonal. For comparison, these states are also considered in Fig. 6.

Remarkable the fact that the angle of the proton and neutron angular momenta in the dipole states given in Figs. 3), 4) and 5), is different from that characterizing the ground band states and shown in Fig. 2 for three sets of the proton and neutron deformation parameters, (d_p, d_n). Note that for the state 0^+, heading the ground band, the two angular momenta, $\vec{J}_p, \vec{J}_n$, are equal in magnitude, have the same direction but different orientation. This property holds irrespective of the deformation parameters d_p, d_n. From the value of 180^0, the angle is decreasing when the total angular momentum is increased. When the proton and neutron deformations are equal the angle tends to zero for J very large. The alignment is reached faster for small than for large deformations. If the deformations are different namely one is small and the other moderately large, the angle is very slowly decreasing for $J \geq 2$, otherwise keeping close to 90^0, reflecting the fact that for small deformation the rotational axis is almost indefinite.

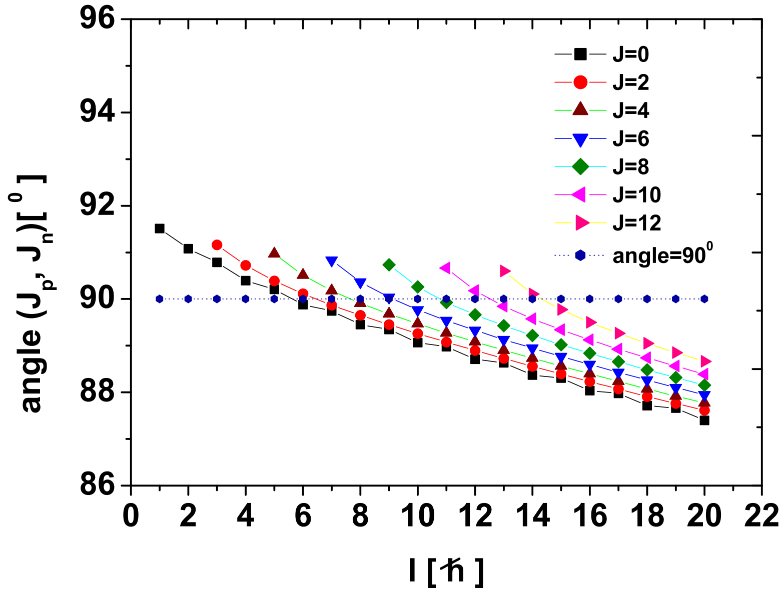

FIG. 6: The angle between $\vec{J}_p$ and $\vec{J}_n$ within the boson dipole state $\Phi^{(2qp;J1)}_{JI;M}(d_p, d_n)$. The deformation parameters are $d_p = 0.2$ and $d_n = 2.4$.

VI. MAGNETIC DIPOLE TRANSITIONS

The magnetic moment of the phenomenological core is defined by:

$$\vec{\mu}_c = g_p \vec{J}_p + g_n \vec{J}_n \equiv g_c \vec{J}_{pn}. \tag{6.1}$$

where g_p, g_n and g_c denote the gyromagnetic factors for proton neutrons and the core. Multiplying this with $\vec{J}_c = \vec{J}_{pn}$, and averaging the result with the function $\Psi^{(2qp;J1)}_{JI;M}$ one obtains an equation determining g_c:

$$g_{c;JI} = \frac{g_p + g_n}{2} + \frac{g_p - g_n}{2} \frac{\widetilde{J}^{(1)}_{p;JI}(\widetilde{J}^{(1)}_{p;JI} + 1) - \widetilde{J}^{(1)}_{n;JI}(\widetilde{J}^{(1)}_{n;JI} + 1)}{\widetilde{J}^{(1)}_{pn;JI}(\widetilde{J}^{(1)}_{pn;JI} + 1)}. \tag{6.2}$$

Note that since the deformation parameters for proton and neutron are equal with each other, the average values of proton and neutron angular momenta are the same, $\widetilde{J}^{(1)}_p = \widetilde{J}^{(1)}_n$ which results in having a simple expression for the core gyromagnetic factor:

$$g_c = \frac{g_p + g_n}{2}. \tag{6.3}$$

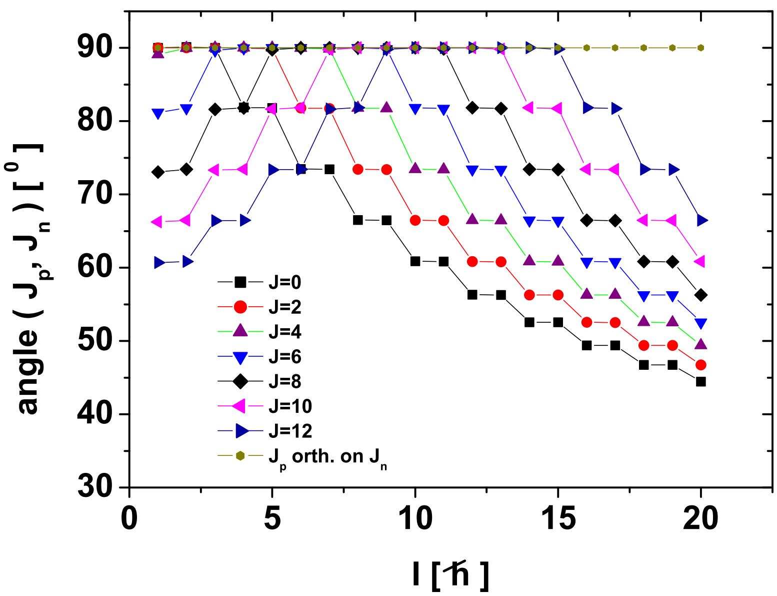

FIG. 7: The angle between $\vec{J}_p$ and $\vec{J}_n$ within the boson dipole state $\Psi^{(2qp;01)}_{JI;M}(d)$. The deformation parameter for protons is equal to that for neutrons and $\rho = 0.2$.

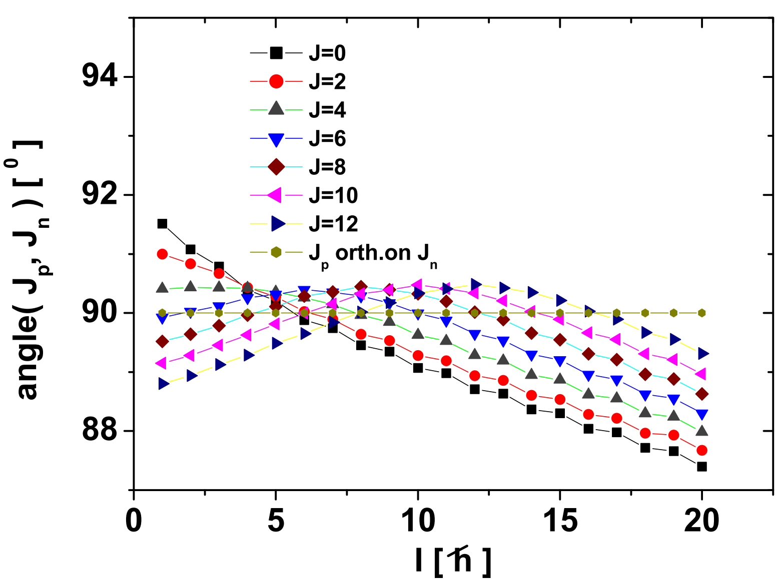

FIG. 8: The angle between $\vec{J}_p$ and $\vec{J}_n$ within the boson dipole state $\Psi^{(2qp;01)}_{JI;M}(d_p, d_n)$. The deformation parameters are $d_p = 0.2$ and $d_n = 2.4$.

The expression 6.2 can be easily derived by expressing first the core magnetic moment as a linear combination of the sum and the difference of proton and neutron angular momenta:

$$\vec{\mu}_c = \frac{g_p + g_n}{2}\left(\vec{J}_p + \vec{J}_n\right) + \frac{g_p - g_n}{2}\left(\vec{J}_p - \vec{J}_n\right). \tag{6.4}$$

Since the scissors state, 1^+, is antisymmetric with respect to the proton neutron permutation while the ground state, 0^+, is symmetric, only the second term from the above equation contributes to the transition $0^+ \to 1^+$. This feature is not preserved when we treat the intra transitions of the chiral band the states participating to the transition behaving similarly at the proton neutron permutation.

Denoting by g_F the gyromagnetic factor for the two quasiparticle factor state and following a similar procedure as above we get for the whole system the following gyromagnetic factor:

$$g_{JI} = \frac{g_F + g_c}{2} + \frac{g_c - g_F}{2}\frac{\widetilde{J}^{(1)}_{pn;JI}(\widetilde{J}^{(1)}_{pn;JI}+1) - J(J+1)}{I(I+1)}. \tag{6.5}$$

We note that both gyromagnetic factors for the core and for the whole system depend on the angular momenta J and I.

In order to calculate the M1 transition probability we need the following reduced matrix elements:

$$\begin{aligned}
\langle\Psi^{(2qp;J1)}_{JI}||J_F||\Psi^{(2qp;J1)}_{JI'}\rangle &= 2\hat{I}'\hat{J}\sqrt{J(J+1)}N_{JI}N_{JI'}\sum_{J_1}\left(N^{(1)}_{J_1}\right)^{-2}\left(C^{J\ J_1\ I}_{J\ 1\ J+1}\right)^2 W(I'J_11J;JI),\\
\langle\Psi^{(2qp;J1)}_{JI}||g_pJ_p + g_nJ_n||\Psi^{(2qp;J1)}_{JI'}\rangle &= N_{JI}N_{JI'}\hat{I}'\hat{1}\sum_{J_1}C^{J\ J_1\ I}_{J\ 1\ J+1}C^{J\ J_1\ I'}_{J\ 1\ J+1}\left(N^{(1)}_{J_1}\right)^{-2}W(JJ_1I1;I'J_1)\\
\times &\left(g_p\sqrt{\widetilde{J}_{p;J_1}(\widetilde{J}_{p;J_1}+1)} + g_n\sqrt{\widetilde{J}_{n;J_1}(\widetilde{J}_{n;J_1}+1)}\right).
\end{aligned} \tag{6.6}$$

Using the previous results regarding the avenge value of $\hat{J}^2_\tau$ the last expression of the above equations, considered for the case $I' = I$, simplifies to:

$$\langle\Psi^{(2qp;J1)}_{JI}||g_pJ_p + g_nJ_n||\Psi^{(2qp;J1)}_{JI}\rangle = g_p\sqrt{\widetilde{J}_{p;JI}(\widetilde{J}_{p;JI}+1)} + g_n\sqrt{\widetilde{J}_{n;JI}(\widetilde{J}_{n;JI}+1)}. \tag{6.7}$$

The M1 transition operator is defined by:

$$M_{1,m} = \sqrt{\frac{3}{4\pi}}\mu_{1,m}. \tag{6.8}$$

In Refs.[7–9] we pointed out a drawback of the phenomenological descriptions of the magnetic states consisting of that the transition operator does not take care of the Hamiltonian model structure, i.e. is independent of the states participating at transition. Therein, we proposed a possible solution for correcting the mentioned drawback.

Indeed, using the classical expression for the magnetic moment:

$$\vec{\mu}_k = \frac{1}{2c}\int \rho_p(\vec{R}\times\vec{v})_k d\vec{r}, \tag{6.9}$$

with ρ_p and $\vec{v}$ denoting the proton charge density and the velocity of an elementary volume of proton matter having the coordinate $\vec{r}$, and integrating on a liquid drop volume whose surface is expressed in terms of the quadrupole coordinates α_μ, one arrives at a quadratic expressions in coordinates and their time derivatives. The coordinates and their conjugate momenta are quantized by:

$$\begin{aligned}\alpha_{p\mu} &= \frac{1}{k_p\sqrt{2}}\left(b^\dagger_{p\mu} + (-)^\mu b_{p,-\mu}\right), \\ \dot{\alpha}_{p\mu} &= \frac{1}{i\hbar}\left[H, \alpha_{p\mu}\right].\end{aligned} \tag{6.10}$$

where "" denotes the time derivative operation. In this way a simple boson expression for the transition operator was obtained:

$$M_{1\mu} = \sqrt{2}\frac{Mc}{\hbar}R_0\mu_N\mathcal{F}_\mu, R_0 = 1.2A^{1/3}. \tag{6.11}$$

where M denotes the proton mass, μ_N the nuclear magneton and c the light velocity. The reduced form-factor $\mathcal{F}_{k_p}$ has the expression:

$$\begin{aligned} q\mathcal{F}_\mu = -\frac{i}{\hbar c k_p^2}\Bigg[&(A_1+6A_4)\hat{J}_{p\mu} + \frac{A_3}{5}\hat{J}_{n\mu} + \frac{\sqrt{10}}{4}(A_2-A_1)\left((b^\dagger_n b^\dagger_p)_{1\mu} + (b^\dagger_n b_p)_{1\mu} + (b^\dagger_p b_n)_{1\mu} - (b_n b_p)_{1\mu}\right) \\ + \sqrt{2}A_3&\left[-\frac{1}{\sqrt{10}}(\Omega^\dagger_n\hat{J}_{p\mu} + \hat{J}_{p\mu}\Omega_n) - \Omega^\dagger_{pn}[-(b^\dagger_p b_n)_{1\mu} + (b_n b_p)_{1\mu}] + [(b^\dagger_n b^\dagger_p)_{1\mu} + (b^\dagger_n b_p)_{1\mu}]\Omega_{np}\right]\Bigg]. \end{aligned} \tag{6.12}$$

Here q stands for the momentum transfer when a transition from an initial state of energy E_i to a final state of energy E_f takes place:

$$q = \frac{E_i - E_f}{\hbar c}. \tag{6.13}$$

From the above equations we note that even in the second order in bosons, the gyromagnetic factors have components different of the angular momenta $\hat{J}_p$ and $\hat{J}_n$ which are proportional to the proton neutron dipole operators. Although the present formalism is purely a phenomenological one and therefore the magnetic moments of neutrons are not included, due to the proton neutron coupling terms from the model Hamiltonian the neutron gyromagnetic factor is not vanishing.

Actually restricting the expression for the transition operator to the angular momenta the above equation provides analytical expressions for the proton and neutron system. For illustration in Table I we give the results of our calculations for the reduced magnetic dipole transitions between two adjacent states from a two quasiparticle band, for two sets of the deformation parameters. These are chosen such that correspond to a near vibrational regime. We recall that a rotational picture is reached for a deformation parameter larger than 3 [20]. We note that for $J \geq 6$, where J denotes the quasiparticle total angular momentum, and system angular momentum I larger than 10, the transitions might be consider of collective nature. Although we trucated the angular momentum I to 20, from Table I it is conspicous that the larger is I the large the M1 strength.

$I \rightarrow (I-1)$	(d_p, d_n)=(1.0, 1.0)							(d_p, d_n)=(0.2, 2.4)						
I	J=0	2	4	6	8	10	12	J=0	2	4	6	8	10	12
2	0.929							0.691						
3	0.720							0.535						
4	0.765	0.057						0.468	0.112					
5	0.669	0.158						0.409	0.248					
6	0.773	0.216	0.169					0.393	0.346	0.367				
7	0.704	0.287	0.438					0.361	0.415	0.786				
8	0.832	0.297	0.648	0.280				0.362	0.463	1.110	0.656			
9	0.773	0.358	0.833	0.722				0.340	0.500	1.353	1.402			
10	0.913	0.335	0.950	1.104	0.376			0.350	0.524	1.538	2.011	0.939		
11	0.858	0.400	1.073	1.437	0.979			0.333	0.547	1.679	2.491	2.014		
12	1.004	0.352	1.131	1.692	1.531	0.459		0.346	0.557	1.789	2.877	2.938	1.204	
13	0.951	0.427	1.224	1.921	2.023	1.206		0.332	0.575	1.876	3.184	3.681	2.593	
14	1.102	0.359	1.242	2.087	2.429	1.916	0.531	0.348	0.576	1.945	3.432	4.301	3.811	1.447
15	1.050	0.446	1.322	2.250	2.787	2.565	1.404	0.335	0.593	2.000	3.635	4.814	4.845	3.130
16	1.204	0.359	1.313	2.356	3.078	3.124	2.259	0.352	0.585	2.044	3.802	5.242	5.721	4.641
17	1.152	0.459	1.388	2.478	3.341	3.622	3.057	0.340	0.603	2.082	3.941	5.601	6.464	5.955
18	1.308	0.356	1.356	2.544	3.550	4.047	3.766	0.359	0.589	2.110	4.057	5.905	7.099	7.094
19	1.257	0.470	1.434	2.641	3.748	4.428	4.406	0.348	0.614	2.140	4.155	6.164	7.644	8.080
20	1.415	0.354	1.383	2.677	3.899	4.751	4.968	0.368	0.609	2.161	4.235	6.382	8.113	8.938

TABLE I: The BM1 values, given in units of μ_N^2, of the transitions $I \rightarrow (I-1)$ calculated with the wave functions $\Psi_{JI;M}^{(2qp;J1)}$ given by Eq.(4.5), for two sets of deformation parameters (d_p, d_n). The magnetic dipole transition operator is determined by the following gyromagnetic factors: $g_F = 1.3527\mu_N$; $g_p = 0.666\mu_N$; $g_n = 0.133\mu_N$.

VII. CONCLUSION

In the presious Sections we formulated a semi-phenomenologicl model for describe the magnetic bands for even even nucleai which almost spherical or moderately deformed. The model Hamiltonian involves a term whci break the chiral symmetry. Due to this term there are four bands which are related by a specific chiral transformation. Energies for these bands are defined as average values of the model Hamiltonian and its chirally transformed ones with a dipole two quaisparticle coupled to a phenomenological boson dipole band. Energies and application to real nuclei will be given in a subsequent publication. We note thet he chiral bands cross the phenomenological boson dipole band and therefore we expect that several backbendings will show up. Our description is different from the ones from literature in the following respects. While the previous formalisms deal with odd-odd nuclei here we treat even-even nuclei. While until now there were only two magnetic bands related by a chiral transformation, here we found four magnetic bands having this property. Here we considered two proton quasiparticle bands but alternatively we could chose two neutron quasiparticle and one proton plus one neutron quasiparticle bands. Of course, the last mentioned band would describe an odd-odd system. We already checked that a two neutron quasiparticle band is characterized by a non-collective M1 transition rate. This feature suggests that, indeed, the orbital magnetic moment carried by protons play an important role in determining a chiral magnetic band.

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VIII. APPENDIX A

Here we give the analytical expression of the model Hamiltonian matrix elements, corresponding to the basis states 4.5:

$$
\begin{aligned}
&\langle\Psi^{(2qp;J1)}_{JI}|H|\Psi^{(2qp;J1)}_{J_1I}\rangle = -4\hat{2}\hat{J}\hat{J}_1X^{(\tau)}_{pc}N^{(2qp;J1)}_{JI}N^{(2qp;J_11)}_{J_1I}\eta^{(-)}_{jj}W(JjJ_1j;j2)\\
&\times\sum_{J'J''}\hat{J}'C^{J\ J'I}_{J\ 1\ J+1}C^{J_1\ J''\ I}_{J_1\ 1\ J_1+1}W(J_12IJ';JJ'')\langle\phi^{(1)}_{J'}||b^\dagger_\tau+b_\tau||\phi^{(1)}_{J''}\rangle\\
&-\ X_{sS}\delta_{J,J_1}\left[I(I+1)-J(J+1)-\left(N^{(2qp;J1)}_{JI}\right)^2\sum_{J'}2J'(J'+1)\left(C^{J\ J'\ I}_{J\ 1\ J+1}\right)^2\left(N^{(1)}_{J'}\right)^2\right],\\
&\langle\phi^{(1)}_{IM}|H|\Psi^{(2qp;J1)}_{JI;M}\rangle = 4X^{(\tau)}_{pc}\xi^{(+)}_{jj}N^{(2qp;J1)}_{JI}\delta_{J,2}\sum_{J'}\left(N^{(1)}_{J'}\right)^{-1}C^{J'\ J\ I}_{1\ J;J+1}\langle\phi^{(1)}_{I}||b^\dagger_\tau+b_\tau||\phi^{(1)}_{J'}\rangle, \tau=p,n\\
&\langle\Psi^{(2qp;J1)}_{JI;M}|H|\phi^{(1)}_{IM}\rangle = \langle\phi^{(1)}_{IM}|H|\Psi^{(2qp;J1)}_{JI;M}\rangle. \qquad (A.1)
\end{aligned}
$$

The notation W(abcd;ef) stands for the Racah coefficients. The isospin quantum number τ takes the values p or n depending on whether the two quasiparticle component is proton or of neutron nature and moreover the model Hamiltonian describes the coupling of the τ-like particles to the core.

We note that the matrix elements of the model Hamiltonian are expressed in terms of the reduced matrix elements of the quadrupole operators between states belonging to the phenomenological dipole band. These are given analytically below:

$$
\begin{aligned}
\langle\phi^{(1)}_{I'}||b_\tau||\phi^{(1)}_{I}\rangle &= \frac{2I+1}{2I'+1}C^{I\ 2\ I'}_{1\ 0\ 1}\frac{N^{(1)}_I}{N^{(1)}_{I'}}+3\hat{I}N^{(1)}_IN^{(1)}_{I'}\sum_{I_1I_2}F^{I'I}_{I_1I_2}C^{I_1\ 1\ I'}_{0\ 1\ 1}\left(N^{(g)}_{I_1}\right)^{-2},\\
F^{I'I}_{I_1I_2} &= \hat{I}_2C^{2\ 1\ I_2}_{0\ 1\ 1}C^{I_1\ I_2\ I'}_{0\ 1\ 1}W(22I_22;11)W(I'2I_1I_2;I1),\\
\langle\phi^{(1)}_{I}||b^\dagger_\tau||\phi^{(1)}_{I'}\rangle &= \frac{\hat{I}'}{\hat{I}}(-1)^{I-I'}\langle\phi^{(1)}_{I'}||b_\tau||\phi^{(1)}_{I}\rangle, \tau=p,n. \qquad (A.2)
\end{aligned}
$$

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